

Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection

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ABSTRACT

The problem of laminar fluid flow which results from the stretching of a vertical surface with variable stream conditions in a nanofluid has been investigated numerically. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis in the presence of magnetic field. The symmetry groups admitted by the corresponding boundary value problem are obtained by using a special form of Lie group transformations viz. scaling group of transformations. An exact solution is obtained for translation symmetry and numerical solutions for scaling symmetry. This solution depends on a Lewis number, magnetic field, Brownian motion parameter and thermophoretic parameter. The conclusion is drawn that the flow field and temperature and nanoparticle volume fraction profiles are significantly influenced by these parameters.

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1. Introduction

A nanofluid is a new class of heat transfer fluids that contain a base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The thermal conductivity of the ordinary heat transfer fluids is not adequate to meet today's cooling rate requirements. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids. One of the possible mechanisms for anomalous increase in the thermal conductivity of nanofluids is the Brownian motions of the nanoparticles inside the base fluids. A variety of nuclear reactor designs featured by enhanced safety and improved economics are being proposed by the nuclear power industry around the world to more realistically solve the future energy supply shortfall. Nanofluid coolants showing an improved thermal performance are being considered as a new key technology to secure nuclear safety and economics. Nanofluids are suspensions of nanoparticles in fluids that show significant enhancement of their properties at modest nanoparticle concentrations. Many of the publications on nanofluids are about understanding their behavior so that they can be utilized where straight heat transfer enhancement is paramount as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food. In nuclear reactors, the heat is removed from fuel elements via forced convection, mak-

ing this a much more important heat transfer process. Although nanofluids exhibit better heat transfer properties than pure substances, they also have a higher viscosity, which corresponds to an increase in pumping power. Nanofluids are engineered colloidal suspensions of nanoparticles in water and exhibit a very significant enhancement of the boiling critical heat flux at modest nanoparticle concentrations. They are also used in other electronic applications which use microfluidic applications.

Magnetic nanofluid is a unique material that has both the liquid and magnetic properties. Many of the physical properties of these fluids can be tuned by varying magnetic field. In addition, they have been wonderful model system for fundamental studies. As the magnetic nanofluids are easy to manipulate with an external magnetic field, they have been used for a variety of studies. Particle transport and deposition in flowing suspensions onto collector surfaces is of importance in a broad field of applications. Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. Nanofluids are suspensions of submicronic solid particles (nanoparticles) in common fluids. The term "nanofluid" refers to a liquid containing a suspension of submicronic solid particles (nanoparticles). The term was coined by Choi (1995). The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. (1993). This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems (Buongiorno and Hu (2005)). A comprehensive survey of convective transport in nanofluids was made by Buongiorno (2006), who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on the further heat transfer enhancement observed in

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Nomenclature

B_0	magnetic field strength
C	nanoparticles volume fraction
C_f	skin-friction coefficient
C_w	nanoparticle volume fraction at the wall
C_∞	ambient nanoparticle volume fraction
c_p	specific heat at constant pressure
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	dimensionless stream function
g	acceleration due to gravity
k	thermal conductive
Le	Lewis number
M	magnetic parameter
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nr	buoyancy ratio
Pr	Prandtl number
P	pressure
Rax	local Rayleigh number
S	suction/injection parameter
T	temperature of the fluid
T_w	temperature at the wall
T_∞	ambient temperature
\vec{v}	velocity vector
u, v	velocity components along x- and y-axes
$U(x)$	uniform velocity of the free stream flow
V_0	velocity of suction/injection

Greek symbols

α	thermal conductivity
β	coefficient of thermal expansion
θ	dimensionless temperature
ϕ	dimensionless nanoparticle volume fraction
η	similarity variable
μ	dynamic viscosity
σ	electric conductivity of the fluid
ρ_f	density of the base fluid
ρ_p	nanoparticle mass density
$(\rho c)_f$	heat capacity of the base fluid
$(\rho c)_p$	effective heat capacity of the nanoparticle material
τ	heat capacity ratio
ν	kinematic viscosity
ψ	stream function

convective situations. Very recently, Kuznetsov and Nield (2009) have examined the influence of nanoparticles on natural convection boundary-layer flow past a vertical plate, using a model in which Brownian motion and thermophoresis are accounted for. The authors have assumed the simplest possible boundary conditions, namely those in which both the temperature and the nanoparticle fraction are constant along the wall. Further, Nield and Kuznetsov (2009) have studied the Cheng and Minkowycz (1977) problem of natural convection past a vertical plate, in a porous medium saturated by a nanofluid. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. For the porous medium the Darcy model has been employed.

In the present study we considered the combined effects of Brownian motion and thermophoresis to get the gradient of nanoparticles' volume fraction. Using Lie group analysis, three-dimensional, unsteady, laminar boundary layer equations of non-Newtonian fluids were studied by Yurusoy and Pakdemirli (1997, 1999). Using Lie group analysis, they obtained two dif-

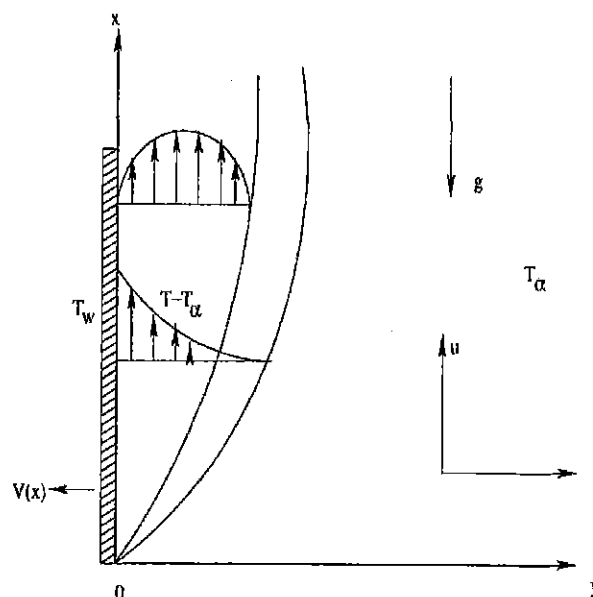


Fig. 1. Physical model of boundary layer flow over a vertical stretching surface.

ferent reductions to ordinary differential equations. They studied the effects of a moving surface with vertical suction or injection through the porous surface and also analyzed the exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. Yurusoy et al. (2001) investigated the Lie group analysis of creeping flow of a second grade fluid. They constructed an exponential type of exact solution using the translation symmetry and a series type of approximate solution using the scaling symmetry and also discussed some boundary value problems. The objective of the present study is to analyze the development of the steady boundary layer flow, heat transfer and nanoparticle volume fraction over a stretching surface in a nanofluid for various parameters using scaling group of transformations viz., Lie group transformations (Fig. 1).

2. Mathematical analysis

We consider a two-dimensional problem. We select a coordinate frame in which the x-axis is aligned vertically upwards. We consider a vertical plate at $y = 0$. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. At this boundary the temperature T and the nanoparticle fraction ϕ take constant values T_w and ϕ_w , respectively. The ambient values, attained as y tends to infinity, of T and ϕ are denoted by T_∞ and ϕ_∞ , respectively.

The Oberbeck–Boussinesq approximation is employed. The following four field equations embody the conservation of total mass, momentum, thermal energy, and nanoparticles, respectively. The field variables are the velocity \vec{v} , the temperature T and the nanoparticle volume fraction ϕ .

$$\nabla \cdot \vec{v} = 0 \quad (1)$$

$$\rho_f \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p - \sigma \vec{B}_0^2 \vec{v} + \mu \nabla^2 \vec{v} + [C\rho_p + (1-C)\{\rho_f(1-\beta(T-T_\infty))\}] \vec{g} \quad (2)$$

$$(\rho C)_f \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = k \nabla^2 T + (\rho C)_p \left[D_B \nabla C \cdot \nabla T + \left(\frac{\nabla T}{T_\infty} \right) \nabla T \cdot \nabla T \right] \quad (3)$$

$$\frac{\partial C}{\partial t} + \vec{v} \cdot \nabla C = D_B \nabla^2 C + \frac{\nabla T}{T_\infty} \cdot \nabla^2 T \quad (4)$$

We write $\vec{v} = (u, v)$.

Here ρ_f is the density of the base fluid, B_0 is a constant magnetic field of strength, σ is electric conductivity, μ , k and β are the density, viscosity, thermal conductivity and volumetric volume expansion coefficient of the nanofluid, while ρ_p is the density of the particles. The gravitational acceleration is denoted by \vec{g} . The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T . Details of the derivation of Eqs. (3) and (4) are given in the paper by Buongiorno (2006) and Nield and Kuznetsov (2009). The boundary conditions are taken to be

$$u = U(x), \quad v = V(x), \quad C = C_w, \quad T = T_w \quad \text{at } y = 0; \\ u = 0, \quad C = C_\infty, \quad T = T_\infty \quad \text{as } y \rightarrow \infty \quad (5)$$

We consider a steady state flow.

In keeping with the Oberbeck–Boussinesq approximation and an assumption that the nanoparticle concentration is dilute, and with a suitable choice for the reference pressure, we can linearize the momentum equation and write Eq. (2) as

$$\rho_f \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p - \sigma B_0^2 \vec{v} + \mu \nabla^2 \vec{v} \\ + [(\rho_p - \rho_f)(C - C_\infty) + (1 - C_\infty)\rho_f\beta(T - T_\infty)] \vec{g} \quad (6)$$

We now make the standard boundary-layer approximation, based on a scale analysis, and write the governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u - \rho_f \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \\ + [(1 - C_\infty)\rho_f\beta g(T - T_\infty) - (\rho_p - \rho_f)g(C - C_\infty)] \quad (8)$$

$$\frac{\partial p}{\partial y} = 0 \quad (9)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (10)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \quad (11)$$

where u and v are the velocity components along the x and y axes, respectively, $\alpha = k/(\rho C)_f$ is the thermal diffusivity of the fluid, ν is the kinematic viscosity coefficient and $\tau = (\rho C)_p/(\rho C)_f$.

The stream wise velocity and the suction/injection velocity are taken as

$$U(x) = cx^m, \quad V(x) = V_0 x^{(m-1)/2} \quad (12)$$

Here $c > 0$ is constant, T_w is the wall temperature, the power-law exponent m is also constant. In this study we take $c = 1$.

One can eliminate p from Eqs. (8) and (9) by cross-differentiation. At the same time one can introduce a stream function ψ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{and} \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (13)$$

So that Eq. (7) is satisfied identically. We are then left with the following three equations.

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} - \nu \frac{\partial^3 \psi}{\partial y^3} = -\frac{\sigma B_0^2}{\rho_f} \frac{\partial \psi}{\partial y} \\ + (1 - \phi_\infty)\rho_{f\infty}\beta g\theta - (\rho_p - \rho_{f\infty})g\phi \Delta\phi \quad (14)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} + \tau \left[D_B \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial \theta}{\partial y} \right)^2 \right] \quad (15)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 \theta}{\partial y^2} \quad (16)$$

$$\frac{\partial \psi}{\partial y} = x^m, \quad \frac{\partial \psi}{\partial x} = -V_0 x^{\frac{m-1}{2}}, \quad \theta = \phi = 1 \quad \text{at } y = 0; \\ \frac{\partial \psi}{\partial y} \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad (17)$$

where $\Delta\theta = T_w - T_\infty$ and $\Delta\phi = C_w - C_\infty$.

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations,

$$\Gamma: x^* = xe^{\epsilon\alpha_1}, \quad y^* = ye^{\epsilon\alpha_2}, \quad \psi^* = \psi e^{\epsilon\alpha_3}, \quad u^* = ue^{\epsilon\alpha_4}, \\ v^* = ve^{\epsilon\alpha_5}, \quad \theta^* = \theta e^{\epsilon\alpha_6}, \quad \phi^* = \phi e^{\epsilon\alpha_7} \quad (18)$$

Eq. (18) may be considered as a point-transformation which transforms co-ordinates $(x, y, \psi, u, v, \theta, \phi)$ to the coordinates $(x^*, y^*, \psi^*, u^*, v^*, \theta^*, \phi^*)$.

Substituting (18) in (14)–(16) we get,

$$e^{\epsilon(\alpha_1+2\alpha_2-2\alpha_3)} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial y^{*2}} \right) = \nu e^{\epsilon(3\alpha_2-\alpha_3)} \frac{\partial^3 \psi^*}{\partial y^{*3}} \\ + e^{-\epsilon\alpha_6} (1 - \phi_\infty)\rho_{f\infty}\beta g\theta \Delta\theta - e^{\epsilon(\alpha_2-\alpha_3)} \frac{\sigma B_0^2}{\rho_f} \frac{\partial \psi^*}{\partial y^*} \\ - e^{-\epsilon\alpha_7} (\rho_p - \rho_{f\infty})g\phi \Delta\phi \quad (19)$$

$$e^{\epsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial \theta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \theta^*}{\partial y^*} \right) = \alpha e^{\epsilon(2\alpha_2-\alpha_6)} \frac{\partial^2 \theta^*}{\partial y^{*2}} \\ + \tau \left[D_B e^{\epsilon(2\alpha_2-\alpha_6-\alpha_7)} \frac{\partial \phi^*}{\partial y^*} \frac{\partial \theta^*}{\partial y^*} + e^{\epsilon(2\alpha_2-2\alpha_6)} \frac{D_T}{T_\infty} \left\{ \frac{\partial \theta^*}{\partial y^*} \right\}^2 \right] \quad (20)$$

$$e^{\epsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial \phi^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \phi^*}{\partial y^*} \right) \\ = D_B e^{\epsilon(2\alpha_2-\alpha_7)} \frac{\partial^2 \phi^*}{\partial y^{*2}} + \frac{D_T}{T_\infty} e^{\epsilon(2\alpha_2-\alpha_6)} \frac{\partial^2 \theta^*}{\partial y^{*2}} \quad (21)$$

The system will remain invariant under the group of transformations Γ , we would have the following relations among the parameters, namely

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 = \alpha_2 - \alpha_3 = -\alpha_6 = -\alpha_7; \\ \alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = 2\alpha_2 - \alpha_6 = 2\alpha_2 - \alpha_6 - \alpha_7 = 2\alpha_2 - 2\alpha_6 \quad \text{and} \\ \alpha_1 + \alpha_2 - \alpha_3 - \alpha_7 = 2\alpha_2 - \alpha_7 = 2\alpha_2 - \alpha_6$$

These relations $3\alpha_2 - \alpha_3 = \alpha_2 - \alpha_3$ give the value $\alpha_2 = 0$

Hence, $\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3$ gives $\alpha_6 = \alpha_7 = 0$, $\alpha_2 = (1/4)\alpha_1 = (1/3)\alpha_3$.

The boundary conditions yield $\alpha_4 = m\alpha_1 = (1/2)\alpha_1$, $\alpha_5 = ((m-1)/2)\alpha_1 = -(1/4)\alpha_1$ (as $m = 1/2$).

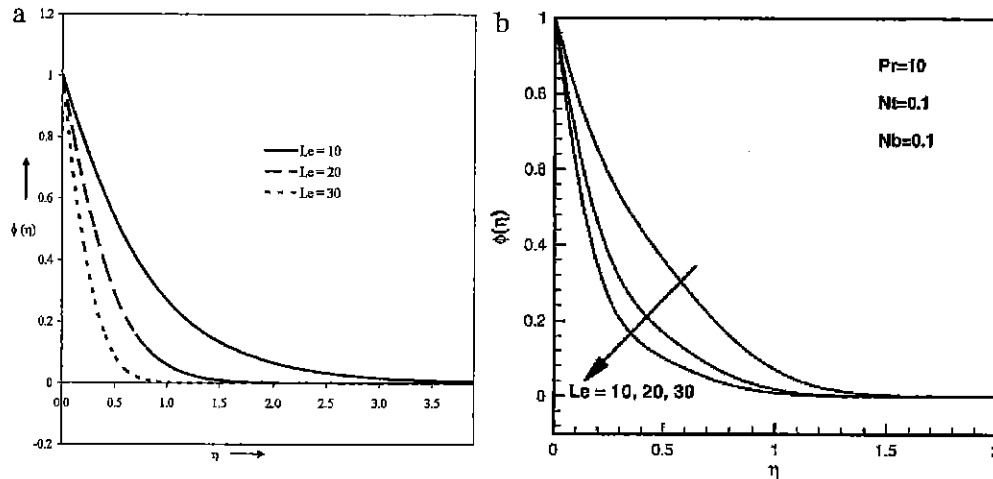


Fig. 2. Comparison of the nanoparticles fraction profiles (present result – (a) with Khan and Pop, 2010 – (b)).

In view of these, the boundary conditions become

$$\frac{\partial \psi^*}{\partial y^*} = x^{*1/2}, \quad \frac{\partial \psi^*}{\partial x} = -V_0 x^{*(-1/4)}, \quad \theta^* = \phi^* = 1 \quad \text{at } y^* = 0$$

$$\text{and } \frac{\partial \psi^*}{\partial y^*} \rightarrow 0, \quad \theta^* \rightarrow 0, \quad \phi^* \rightarrow 0 \quad \text{as } y^* \rightarrow \infty \quad (22)$$

The set of transformations F^* reduces to $x^* = x e^{\varepsilon \alpha_1}$, $y^* = y e^{\varepsilon(\alpha_1/4)}$, $\psi^* = \psi e^{\varepsilon(3\alpha_1/4)}$, $u^* = u e^{\varepsilon(\alpha_1/2)}$, $v^* = v e^{\varepsilon(\alpha_1/4)}$, $\theta^* = \theta$, $\phi^* = \phi$

Expanding by Taylor's method in powers of ε and keeping terms up to the order ε we get

$$x^* - x = x \varepsilon \alpha_1, \quad y^* - y = y \varepsilon \frac{\alpha_1}{4}, \quad \psi^* - \psi = \psi \varepsilon \frac{3\alpha_1}{4},$$

$$u^* - u = u \varepsilon \frac{\alpha_1}{2}, \quad v^* - v = -v \varepsilon \frac{\alpha_1}{4}, \quad \theta^* - \theta = \phi^* - \phi = 0$$

From the above equations we get,

$$y^* x^{*(1/4)} = \eta, \quad \psi^* = x^{*(3/4)} f(\eta), \quad \theta^* = \theta(\eta), \quad \phi^* = \phi(\eta) \quad (23)$$

With the help of these relations, Eqs. (19)–(21) become

$$f''' + \frac{1}{Pr} [0.75ff'' - 0.5f'^2] + Rax [\theta - Nr\phi] + Mf' = 0 \quad (24)$$

$$\theta'' + 0.75f\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0 \quad (25)$$

$$\phi'' + 0.75Lf\phi' + \frac{Nt}{Nb}\theta'' = 0 \quad (26)$$

The boundary conditions take the following form

$$f' = 1, \quad f = -\frac{4V_0}{3}, \quad \theta = \phi = 1 \quad \text{at } \eta = 0 \quad \text{and } f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (27)$$

where $Pr = \nu^*/\alpha^*$ is the Prandtl number, $Rax = ((1 - \phi_\infty)\beta g \Delta \theta)/(\nu^* \alpha^*)$ is the local Rayleigh number, $Nr = ((\rho_p - \rho_\infty)\Delta \phi)/(\rho_\infty \beta \Delta \theta(1 - \phi_\infty))$ is the buoyancy ratio, $Nb = ((\rho c)_p D_B \Delta \phi)/(\alpha(\rho c)_f)$ is the Brownian motion parameter, $Nt = ((\rho c)_p D_T \Delta \theta)/(\alpha(\rho c)_f T_\infty)$ is the thermophoresis parameter, $M = (\sigma B_0^2 U)/\rho_f$ is the magnetic parameter and $Le = \nu^*/D$ is the Lewis number.

The boundary conditions take the following forms.

$$f' = 1, \quad f = S, \quad \theta = \phi = 1, \quad \text{at } \eta^* = 0 \quad \text{and } f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as } \eta^* \rightarrow \infty \quad (28)$$

where $S = -(4/3)V_0$, $S > 0$ corresponds to suction and $S < 0$ corresponds to injection whereas V_0 is the velocity of suction if $V_0 < 0$ and injection if $V_0 > 0$.

3. Numerical solution

The set of non-linear ordinary differential equations (24)–(26) with boundary conditions (28) have been solved by using the Runge–Kutta–Gill algorithm (Gill, 1951) with a systematic guessing of $f'(0)$, $\theta'(0)$ and $\phi'(0)$ by the Shooting Technique with until the boundary conditions at infinity $f(0)$, $\theta(0)$ and $\phi(0)$ decay exponentially to zero. The step size $\Delta \eta = 0.001$ is used while obtaining the numerical solution with η_{\max} , and accuracy to the fifth decimal place is sufficient for convergence. The computations were done by a program which uses a symbolic and computational computer language Matlab. A step size of $\Delta \eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of η_∞ was found to each iteration loop by assignment statement $\eta_\infty = \eta_\infty + \Delta \eta$. The maximum value of η_∞ to each group of parameters Pr , Le , M , Nr , Nb and Nt , determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7} . Effects of development of the steady boundary layer flow, heat transfer and nanoparticle volume fraction over a stretching surface in a nanofluid are studied for different values of Brownian motion parameter, thermophoresis parameter, magnetic parameter and Lewis number. In the following section, the results are discussed in detail.

4. Results and discussion

Numerical analysis are carried out for $0.5 \leq Nb \leq 2.5$, $0.5 \leq Nt \leq 2.0$, $0.5 \leq M \leq 2.0$ and $-2.0 \leq S \leq 1.0$. Eqs. (24)–(26) subject to the boundary conditions (28) have been solved numerically for some values of the governing parameters Pr , Le , M , Nr , Nb and Nt using Runge–Kutta–Gill algorithm with shooting technique. Neglecting the effects of Nb and Nt numbers, the results for the reduced Nusselt number $-\theta'(0)$ are compared with those obtained by Khan and Pop (2010), Wang (1989), and Gorla and Sidawi (1994) for different values of Pr in Table 1. We notice that the comparison shows good agreement for each value of Pr . Therefore, we are confident that the present results are very accurate.

In the absence of Local Rayleigh number Rax , in order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The nanoparticle fraction profiles for Lewis number Le are compared

Table 1

Comparison of results for $-\theta'(0)$ with previous published works.

Pr	Khan and Pop (2010)	Wang (1989)	Gorla and Sidawi (1994)	Present work
0.07	0.0663	0.0656	0.0656	0.066129
0.20	0.1691	0.1691	0.1691	0.169136
0.70	0.4539	0.4539	0.5349	0.454285
2.00	0.9113	0.9113	0.9113	0.911423
7.00	1.8954	1.8954	1.8905	0.895264
20.0	3.3539	3.3539	3.3539	3.353853
70.0	6.4621	6.4622	6.4622	6.462189

with the available exact solution of Khan and Pop (2010), are shown in Fig. 2a and b. It is observed that the agreements with the theoretical solution of nanoparticle fraction profiles are excellent. For a given Nb and Nt , it is clear that there is a fall in nanoparticle fraction with increasing the Lewis number. This is due to the fact that there would be a decrease of nanoparticle volume fraction boundary layer thickness with the increase of Lewis number as one can see from Fig. 2a and b by comparing the curves with $Le = 10$, $Le = 20$ and $Le = 30$.

Volume fraction of nanoparticles is a key parameter for studying the effect of nanoparticles on flow fields and temperature distributions. Thus Figs. 3 and 4 are prepared to present the effect of Brownian motion on temperature distribution and volume fraction of nanoparticle. Figs. 3 and 4 illustrate the typical temperature and nanoparticle volume fraction profiles for various values of Brownian motion parameter, Nb . Temperature of the fluid increases and the nanoparticle volume fraction decreases with increase of Nb . It is interesting to note that Brownian motion of nanoparticles at the molecular and nanoscale levels is a key nanoscale mechanism governing their thermal behavior. In nanofluid systems, due to the size of the nanoparticles Brownian motion takes place which can affect the heat transfer properties. As the particle size scale approaches to the nano-meter scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in heat transfer.

Figs. 5 and 6 display the effect of thermophoretic parameter Nt on temperature and nanoparticle volume fraction profiles. It is to note that the temperature of the fluid increases whereas the nanoparticle volume fraction decreases with increase of Nt . We notice that, positive Nt indicates a cold surface, but is negative to a hot surface. For hot surfaces, thermophoresis tends to blow the nanoparticle volume fraction boundary layer away from the surface since a hot surface repels the sub-micron sized particles

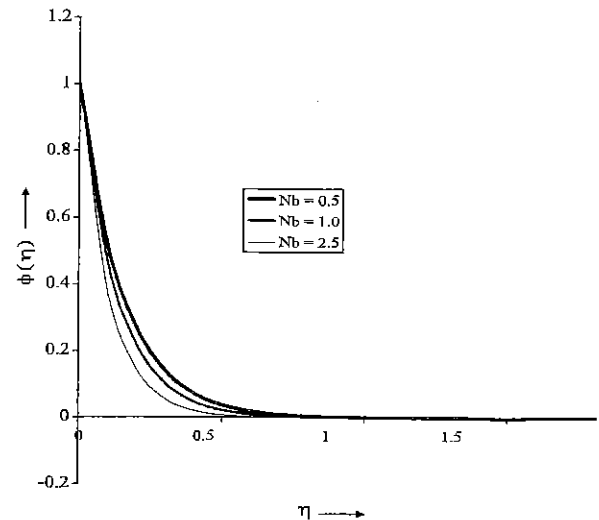


Fig. 4. Effect of Nb over the nanoparticle volume fraction profiles when $Pr = 2.0$, $Le = 3.0$, $Nt = 1.0$, $S = 2.0$ and $Nr = 0.5$, $M = 1.0$.

from it, thereby forming a relatively particle-free layer near the surface. As a consequence, the nanoparticle distribution is formed just outside. In particular, the effect of increasing the thermophoretic parameter Nt is limited to increasing slightly the wall slope of the nanoparticle volume fraction profiles but decreasing the nanoparticle volume fraction. This is true only for small values of Lewis number for which the Brownian diffusion effect is large compared to the convection effect. However, for large values of Lewis number, the diffusion effect is minimal compared to the convection effect

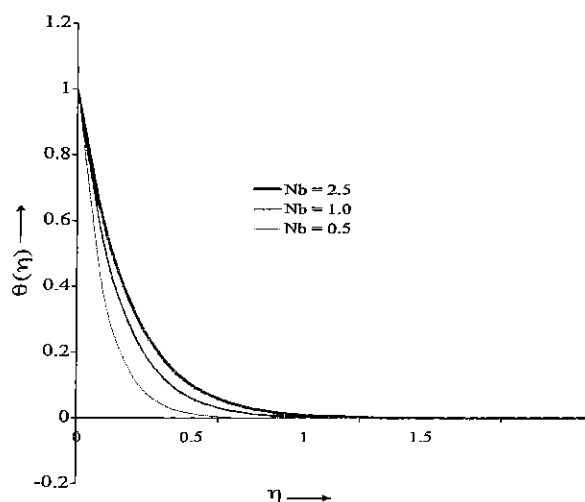


Fig. 3. Temperature profiles for various values of Nb when $Pr = 2.0$, $Le = 3.0$, $Nt = 1.0$, $S = 2.0$ and $Nr = 0.5$, $M = 1.0$.

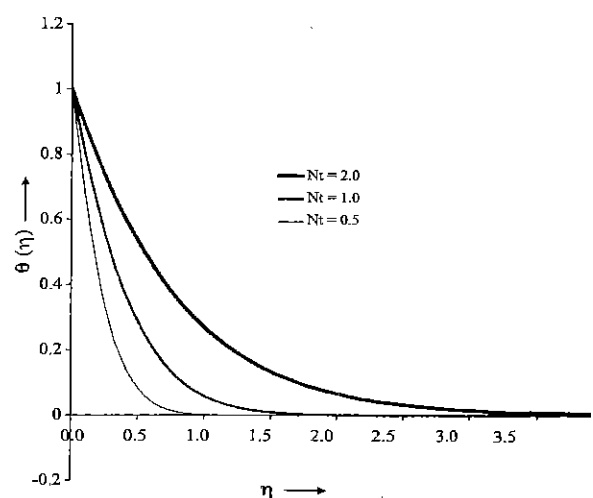


Fig. 5. Temperature profiles for various values of Nt when $Pr = 2.0$, $Le = 3.0$, $Nb = 1.0$, $S = 2.0$ and $Nr = 0.5$, $M = 1.0$.

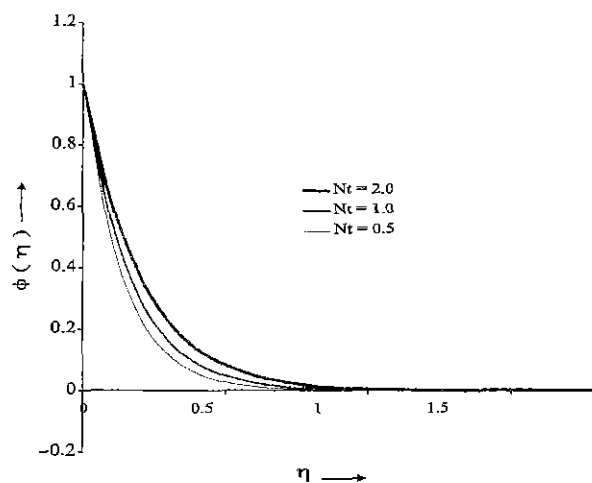


Fig. 6. Effect of Nt over the nanoparticle volume fraction profiles when $Pr = 2.0$, $Le = 3.0$, $Nb = 1.0$, $S = 2.0$ and $Nr = 0.5$, $M = 1.0$.

and, therefore, the thermophoretic parameter Nt is expected to alter the nanoparticle volume fraction boundary layer significantly. Although thermophoresis effect is important in natural convection of nanofluids, there are other parameters that may have effect, and should be considered. These effects include increase in effective viscosity of nanofluids due to the presence of nanoparticles and density variation due to variable volume fraction. More volume fraction of nanoparticles makes nanofluid much viscous and the mixture's convection takes place weaker, thus natural convective Nusselt number decreases due to high viscous fluid. On the other hand it is showed that the separation factor for common nanofluids is positive and density variation due to variable volume fraction of nanoparticles, called particulate buoyancy force, helps nanofluid to have strong convection heat transfer.

Figs. 7–9 depict the influence of the suction/injection parameter S on the velocity, temperature and nanoparticle volume fraction profiles in the boundary layer when the thermophoretic particle deposition is uniform, i.e. $Nt = 1.0$. With the increasing value of the suction ($S > 0$), the velocity is found to decrease (Fig. 7), i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. In case of suction, the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of the Brownian motion effects. This effect acts to

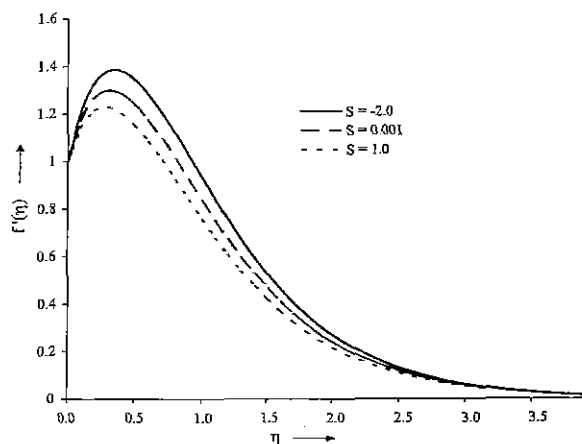


Fig. 7. Effect of suction injection over the velocity profiles when $Pr = 2.0$, $Le = 3.0$, $Nb, Nt = 1.0$ and $Nr = 0.5$, $M = 1.0$.

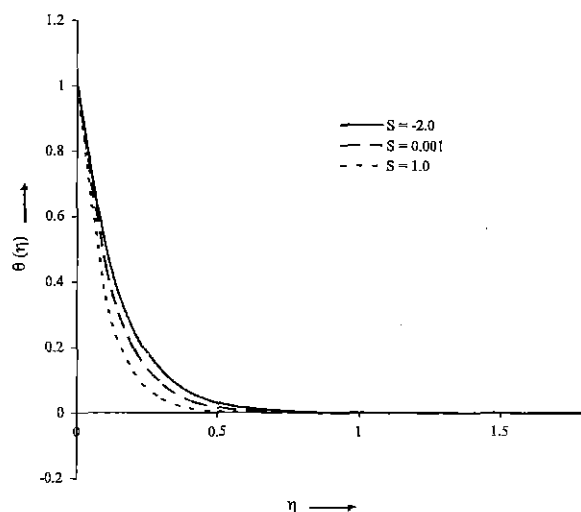


Fig. 8. Effect of suction injection over the temperature profiles when $Pr = 2.0$, $Le = 3.0$, $Nb, Nt = 1.0$ and $Nr = 0.5$, $M = 1.0$.

decrease the wall shear stress. Figs. 8 and 9 exhibit that the temperature $\theta(\eta)$ and nanoparticle volume fraction $\phi(\eta)$ in boundary layer also decrease with the increasing suction parameter ($S > 0$) (the fluid is of uniform thermophoretic particle deposition, i.e. $Nt = 1.0$) ($Pr = 2.0$ and $Nb = 1.0$). The explanation for such behavior is that the fluid is brought closer to the surface and reduces the thermal and nanoparticle volume boundary layer thickness in case of suction. As such then the presence of wall suction decreases velocity boundary layer thicknesses but decreases the thermal and nanoparticle volume fraction boundary layers thickness, i.e. thins out the thermal and nanoparticle volume fraction boundary layers. However, the exact opposite behavior is produced by imposition of wall fluid blowing or injection. These behaviors are also clear from Figs. 7–9. The samples of velocity, temperature and the nanoparticle volume fraction profiles are given in Figs. 7, 8 and 9, respectively. These profiles satisfy the far field boundary conditions (28) asymptotically, which support the numerical results obtained.

Figs. 10 and 11 present typical profiles for velocity and temperature for different values of magnetic parameter. Due to the

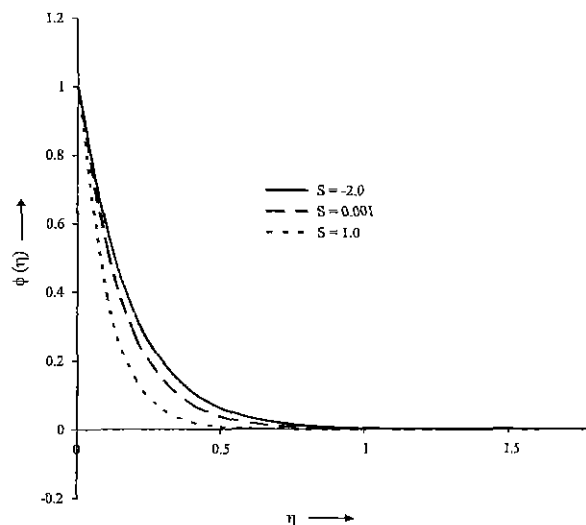


Fig. 9. Effect of suction injection over the nanoparticle volume fraction profiles when $Pr = 2.0$, $Le = 3.0$, $Nb, Nt = 1.0$ and $Nr = 0.5$, $M = 1.0$.

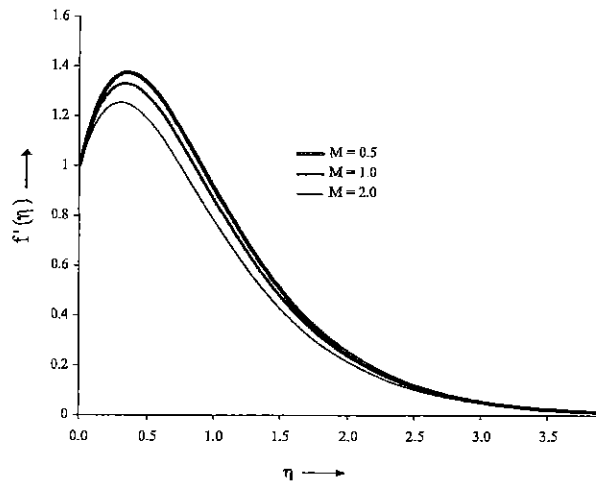


Fig. 10. Magnetic effect over the velocity profiles when $Pr = 2.0$, $Le = 3.0$, $Nb = 1.0$, $S = 2.0$, and $Nr = 0.5$.

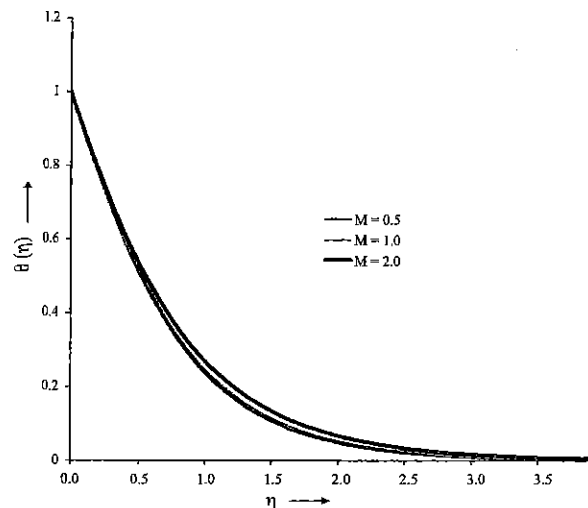


Fig. 11. Magnetic effect over the temperature profiles when $Pr = 2.0$, $Le = 3.0$, $Nb = 1.0$, $S = 2.0$, and $Nr = 0.5$.

uniform thermophoresis particle deposition, it is clearly shown that the velocity of the fluid decreases and the temperature of the fluid increases whereas the nanoparticle volume fraction of the fluid is not significant with increase of the strength of magnetic field. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid and to increase its temperature profiles. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the natural convection flow. Application of a magnetic field moving with the free stream has the tendency to induce a motive force which decreases the motion of the fluid and increases its boundary layer.

5. Conclusions

In the present paper, we have studied theoretically the problem of steady MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection. The model used for the nanofluid incorporates the effects of Brownian

motion and thermophoresis. It is found that the volume fraction of nanoparticles is a key parameter for studying the effect of nanoparticles on flow fields and temperature distributions. It is interesting to note that the impact of thermophoresis particle deposition in the presence of magnetic field with Brownian motion have a substantial effect on the flow field and, thus, on the heat transfer and nanoparticle volume fraction rate from the sheet to the fluid. Particularly, the temperature of the fluid increases whereas the nanoparticle volume fraction decreases with increase of Brownian motion and thermophoretic parameter, respectively. Brownian motion of nanoparticles at the molecular and nanoscale levels is a key nanoscale mechanism governing their thermal behavior. In nanofluid systems, due to the size of the nanoparticles Brownian motion and thermophoresis takes place which can affect the heat transfer properties.

The analysis has helped engineers understand the mechanisms that are most important in the deposition process. Free convective flow through porous media is an area of research undergoing rapid growth in the fluid mechanics and heat transfer and nanoparticle volume fraction field due to its broad range of scientific and engineering applications. One of the technological applications of nanoparticles that hold enormous promise is the use of heat transfer fluids containing suspensions of nanoparticles to confront cooling problems in thermal systems. Hence, the subject of nanofluids is of great interest worldwide for basic and applied research. Nanofluids are important because they can be used in numerous applications involving heat transfer and other applications such as in detergency. Colloids which are also nanofluids have been used in the biomedical field for a long time, and their use will continue to grow. Nanofluids have also been demonstrated for use as smart fluids.

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